Quaternion Space-time (Part 4):

Summary: Here we discuss gluon symmetry and show that a broken version of the $SU(3)$ symmetry which we can call $\approx SU(3)$ allowed the emergence of neutrons. We also introduce a mechanism for the nuclear strong force not dependent on pion exchange. We also show that an expanded symmetry $SU(4)$ accounts nicely for hypercharge.

Gell-Mann Matrices

The Gell-Mann matrices are defined as follows:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Their multiplication table is:

$\times$	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
λ_1	I_a	$i\lambda_3$	$-i\lambda_2$	$*$	$*$	$*$	$*$	$\lambda_1/\sqrt{3}$
λ_2	$-i\lambda_3$	I_a	$i\lambda_1$	$*$	$*$	$*$	$*$	$\lambda_2/\sqrt{3}$
λ_3	$i\lambda_2$	$-i\lambda_1$	I_a	$*$	$*$	$*$	$*$	$\lambda_3/\sqrt{3}$
λ_4	$*$	$*$	$*$	I_b	$*$	$*$	$*$	$*$
λ_5	$*$	$*$	$*$	$*$	I_b	$*$	$*$	$*$
λ_6	$*$	$*$	$*$	$*$	$*$	I_c	$*$	$*$
λ_7	$*$	$*$	$*$	$*$	$*$	$*$	I_c	$*$
λ_8	$\lambda_1/\sqrt{3}$	$\lambda_2/\sqrt{3}$	$\lambda_3/\sqrt{3}$	$*$	$*$	$*$	$*$	α

where $I_a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $I_b = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $I_c = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

The products indicated by '$*$' are those which cannot be expressed as a multiple (real or imaginary) of one of the λs.

Using only the block containing $\{\lambda_1, \lambda_2, \lambda_3\}$ and including I_a we have the multiplication table

$\times$	I_a	λ_1	λ_2	λ_3
I_a	I_a	λ_1	λ_2	λ_3
λ_1	λ_1	I_a	$i\lambda_3$	$-i\lambda_2$
λ_2	λ_2	$-i\lambda_3$	I_a	$i\lambda_1$
λ_3	λ_3	$i\lambda_2$	$-i\lambda_1$	I_a

We recall the multiplication table for the Pauli matrices:

$\times$	I	σ_1	σ_2	σ_3
I	I	σ_1	σ_2	σ_3
σ_1	σ_1	I	$i\sigma_3$	$-i\sigma_2$
σ_2	σ_2	$-i\sigma_3$	I	$i\sigma_1$
σ_3	σ_3	$i\sigma_2$	$-i\sigma_1$	I

So, there is an isomorphism $(I_a, \lambda_1, \lambda_2, \lambda_3) \leftrightarrow (I, \sigma_1, \sigma_2, \sigma_3)$

and since $(\mathbf{1}, \mathbf{i}, \mathbf{j}, \mathbf{k}) \leftrightarrow (I, -i\sigma_1, -i\sigma_2, -i\sigma_3)$

we also have the isomorphism(s)

$(\mathbf{1}, \mathbf{i}, \mathbf{j}, \mathbf{k}) \leftrightarrow (I_a, -i\lambda_1, -i\lambda_2, -i\lambda_3)$ and $(\mathbf{1}, i\mathbf{i}, i\mathbf{j}, i\mathbf{k}) \leftrightarrow (I_a, \lambda_1, \lambda_2, \lambda_3)$

We can therefore write

$\times$	I_a	λ_1	λ_2	λ_3
I_a	I_a	λ_1	λ_2	λ_3
λ_1	λ_1	$\mathbf{1}$	$-\mathbf{k}$	$\mathbf{j}$
λ_2	λ_2	$\mathbf{k}$	$\mathbf{1}$	$-\mathbf{i}$
λ_3	λ_3	$-\mathbf{j}$	$\mathbf{i}$	$\mathbf{1}$

It is then clear that any group which contains the λs as a subset must also contain the quaternion group $\{\pm\mathbf{1}, \pm\mathbf{i}, \pm\mathbf{j}, \pm\mathbf{k}\}$ as a subgroup.

Quantum Chromodynamics (QCD) posits the existence of six color states for quarks: red (r), anti-red ($\overline{r}$), blue (b), anti-blue ($\bar{b}$), green (g) and anti-green ($\overline{g}$).

We can form color-anti-color pairs as follows:

$\times$	$\overline{r}$	$\overline{b}$	$\overline{g}$
r	$r\overline{r}$	$r\overline{b}$	$r\overline{g}$
b	$b\overline{r}$	$b\overline{b}$	$b\overline{g}$
g	$g\overline{r}$	$g\overline{b}$	$g\overline{g}$

Gluons do not exist in singlet states but in eight states of superposition such as:

$$\frac{1}{\sqrt{2}}(r\overline{b}+b\overline{r}) \sim \lambda_1,\ \frac{i}{\sqrt{2}}(-r\overline{b}+b\overline{r}) \sim \lambda_2,\ \frac{1}{\sqrt{2}}(r\overline{r}-b\overline{b}) \sim \lambda_3$$

$$\frac{1}{\sqrt{2}}(r\overline{g}+g\overline{r}) \sim \lambda_4,\ \frac{i}{\sqrt{2}}(-r\overline{g}+g\overline{r}) \sim \lambda_5,\ \frac{1}{\sqrt{2}}(b\overline{g}+g\overline{b}) \sim \lambda_6$$

$$\frac{i}{\sqrt{2}}(-b\overline{g}+g\overline{b}) \sim \lambda_7,\ \frac{1}{\sqrt{6}}(r\overline{r}+b\overline{b}-2g\overline{g}) \sim \lambda_8$$

where $\frac{1}{\sqrt{2}}$ is a normalizing factor.

Notice that the product $\lambda_8\lambda_8 = \alpha = -\frac{1}{3}I_a + \frac{2}{3}I_b + \frac{2}{3}I_c$ has the structure of the proton. I_a, I_b, I_c exist in mutually exclusive color states where I_a plays the role of the down quark (d) with charge -1/3 and I_b and I_c play the role of the up quarks (uu) each with charge +2/3. I_a is in the superimposed color state $\frac{1}{\sqrt{2}}(r\overline{r}+b\overline{b})$, I_b the state $\frac{1}{\sqrt{2}}(r\overline{r}+g\overline{g})$ and I_c the state $\frac{1}{\sqrt{2}}(b\overline{b}+g\overline{g})$.

We transform λ_8 by $r\overline{r}+b\overline{b}-2g\overline{g} \to r\overline{r}+b\overline{b}+i\sqrt{2}g\overline{g}$. This is equivalent to

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1/2 \end{pmatrix} \lambda_8^2 = \frac{2}{3}I_a - \frac{1}{3}I_b - \frac{1}{3}I_c$ which has the form of a neutron.

We set $\lambda_8'^2 = \frac{2}{3}I_a - \frac{1}{3}I_b - \frac{1}{3}I_c = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$.

Similarly to the proton, I_a, I_b, I_c exist in mutually exclusive color states where I_a plays the role of the up quark (u) with charge 2/3 and I_b and I_c play the role of the down quarks (dd) each with charge -1/3. As above, I_a is in the superimposed color state $\frac{1}{\sqrt{2}}(r\overline{r}+b\overline{b})$, I_b the state $\frac{1}{\sqrt{2}}(r\overline{r}+g\overline{g})$ and I_c the state $\frac{1}{\sqrt{2}}(b\overline{b}+g\overline{g})$.

So, $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1/2 \end{pmatrix} p = n$ and $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} n = p$

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