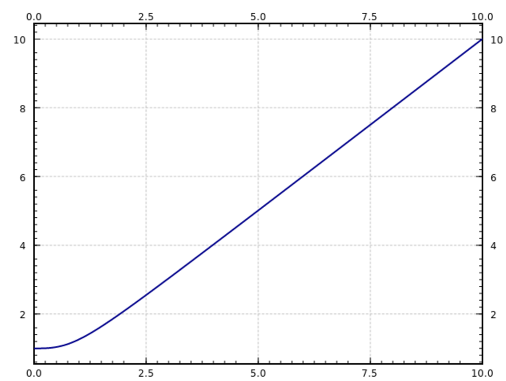
Metric Corrections

Textbook definitions of the metrics of solutions of the GR field equations contain errors which need to be corrected in order to do serious work. Schwarzschild defined his area radius as $R = (r^3+\alpha^3)^{1/3}$. Textbooks replace R with r, the scalar radius making space flat around a gravitating body. The corrected R value makes space hyperbolic around a gravitating body. This error is carried over into the Kerr metric which needs to be corrected there as well.

The corrected Schwarzschild metric is:

$$ds^2 = (1 - \alpha/R)dt^2 - (1 - \alpha/R)^{-1}dR^2 - R^2d\Omega^2$$

where c is set at 1, t is the elapsed time of a clock 'at infinity', r is the scalar distance, α is the radius $2GM$, $R = (r^3 + \alpha^3)^{1/3}$ and $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$



The above shows R as a function of r.

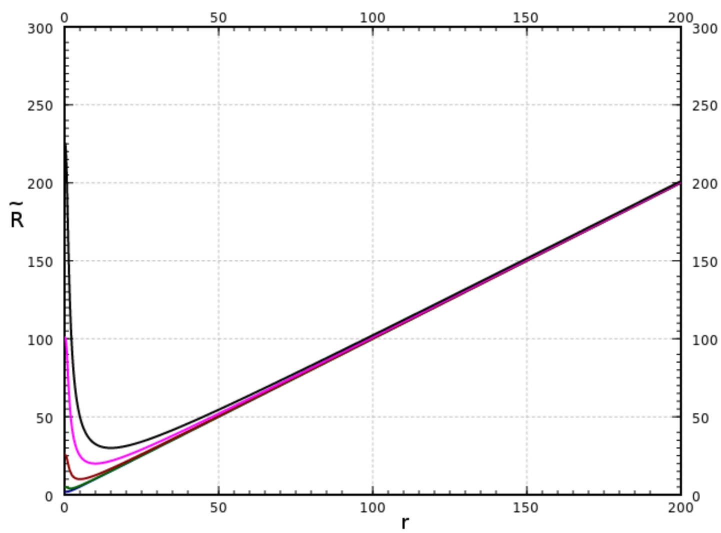
The same analysis as in *The Schwarzschild Metric* can be carried out for rotating gravitating bodies using the Kerr metric which is the axially symmetric solution around a rotating gravitating body with angular momentum J:

The corrected Kerr metric is:

$$ds^2 = (1 - \alpha/\tilde{R})dt^2 + \frac{2\alpha a sin^2\theta}{\tilde{R}} dt d\phi - \frac{\Sigma}{\Delta} dR^2 - d\Omega^2$$

where c is set at 1, t is the elapsed time of a clock 'at infinity', r is the scalar distance, $R = (r^3 + \alpha^3)^{1/3}$, $\alpha = 2GM$, $\tilde{R} = \frac{R^2 + a^2 cos^2\theta}{R}$, $a = J/M$, $\Sigma = R^2 + a^2 cos^2\theta$, $\Delta = R^2 - \alpha R + a^2$, and

$$d\Omega^2 = \Sigma d\theta^2 + (R^2 + a^2 + \frac{\alpha a^2 sin^2\theta}{\tilde{R}}) sin^2\theta d\phi^2$$



The above shows $\tilde{R}$ as a function of r for $sin\theta = 1/2$ and various values of $|a|$.

The gravitational "force" is in the direction of decreasing potential. For the Kerr metric, gravity is repulsive for $cos\theta > \frac{\alpha}{|a|}$. This can only happen if the body is spinning fast enough that $\frac{\alpha}{|a|} < 1$.

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